TONBRIDGE SCHOOL

Scholarship Examination 2005

MATHEMATICS II

Wednesday 4th May 2005
2.00 p.m.

Time allowed: 1 hour 30 minutes

Answer as many questions as you can.
All the questions carry equal marks.

All answers must be supported by adequate explanation.
Calculators may be used in any question.

1. If a solid hemisphere with radius r is split into two identical pieces, the volume (V) and surface area (S) of each piece are given by the formulae:

$V = \frac{1}{3}\pi r^3$ and $S = 2\pi r^2$.

a) Find r if $V = 1000$ cm^3.

b) Find V if S = 500 cm^2.

c) Use algebra to simplify $\frac{S^3}{V^2}$ and hence find the whole number K such that $\frac{S^3}{V^2} = K\pi$.

2. a) A pair of tetrahedral dice each have faces labelled 1, 2, 3, 4. They are thrown onto a desk and the numbers on their bases added to get a total score. Copy and complete the table below showing the probability of each total score.

Total score	2	3	4	5	6	7	8
Probability			$\frac{3}{16}$				

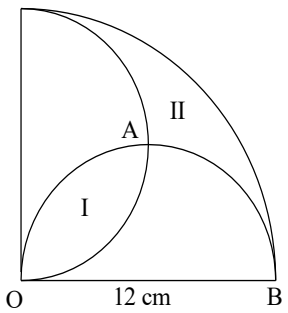
b) A second pair of tetrahedral dice is thrown. One is labelled 1, 2, 2, 3; the other is labelled 1, 3, 3, 5. Make a table as in (a) showing the probability of each total score.

c) Comment on your answers to (a) and (b).

d) i) What is the probability that the two dice in (a) land with the same numbers on their bases?

ii) What is the probability that the two dice in (b) land with the same numbers on their bases?

3. The diagram below shows a large quarter-circle with radius 12 cm and two smaller semicircles each of radius 6 cm.



a) Find the area of region I correct to 2 decimal places.
(Hint: AOB is a right-angled isosceles triangle.)

b) Find the area of region II.

4. The box below shows the four lines in a boy's attempt at solving the equation $12 + 5x = 20 + 3x$.

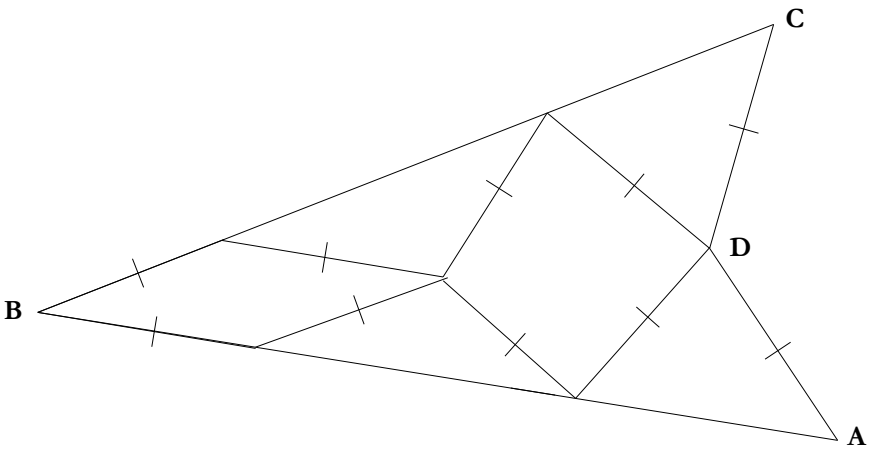
1.	$12 + 5x = 20 + 3x$
2.	$12 + 8x = 20$
3.	$8x = 32$
4.	$x = 4$

a) Identify the mistake(s) in the boy's solution.

b) Give a correct solution. What do you notice about your answer?

c) Find the value of the number C such that the boy's method also gives the correct answer to $C + 6x = 20 + 3x$.

5. The figure below is symmetrical about the line BD and all the marked lengths are equal.

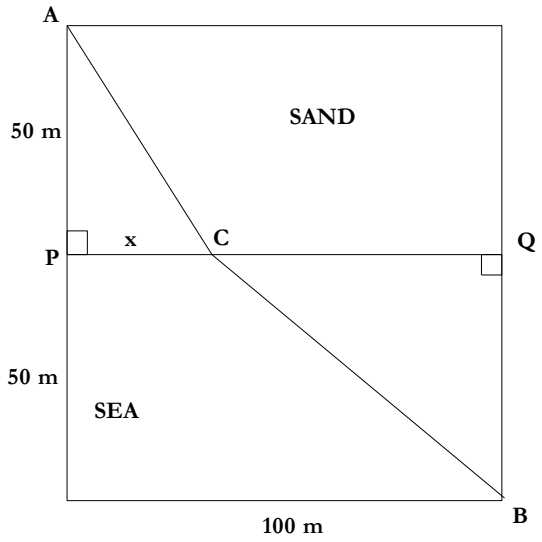


a) If angle ABC is 20°, find angle ADC.

b) In general, find the connection between angle ABC and angle ADC. Justify your answer.

c) If angle ADC is 180°, find angle ABC.

6. The plan below shows a 100 m by 100 m square at the seaside. The top 50 m is sand, the bottom 50 m is sea.



A man at A notices a swimmer in distress at B. He is able to run on the sand from A to C At 3 m/s and he is able to wade through the sea from C to B at 2 m/s. Triangles APC and CQB are right-angled at P and Q respectively; distance PC is x metres.

a) Show that, when $x = 20$ m, the time the man takes to go from A to B (via C) is 65.120 seconds.

b) Use trial and improvement to find the value of x, correct to the nearest metre, that minimises the time the man takes to go from A to B (via C). For each trial value of x, calculate the time taken to 3 decimal places, as in (a).

7. This question concerns the table of numbers below.

- C is the value of A multiplied by B;
- D is given by multiplying all the whole numbers from A to B and adding one;
- E is the square root of D.

	A	***B***	***C***	***D***	***E***
Row 1	1	4	4	25	5
Row 2	2	5	10	121	11
Row 3	-	-	18	361	-
Row 4	-	-	-	-	-

a) Copy this table and fill in the missing entries.

b) Write down the formulae in terms of n for the values of A, B, C, D, E corresponding to Row n.

c) Why are the values of D all odd numbers?

d) Use your formulae in (b) to predict the square root of 10000 x 10001 x 10002 x 10003.